

Problema

Suma finita

Determinar el número natural n tal que:

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{n}+\sqrt{n+1}} = 10$$

En primer lugar, se racionalizan los primeros sumandos (la calculadora lo hace de forma automática):

| Sumando 1 | Sumando 2 | Sumando 3 | Sumando 4 |
|---------------------------------|--|---------------------------------------|--|
| $\frac{1}{1+\sqrt{2}}$ -1+√2 | $\frac{1}{\sqrt{2}+\sqrt{3}}$ √3-√2 | $\frac{1}{\sqrt{3}+\sqrt{4}}$ 2-√3 | $\frac{1}{\sqrt{4}+\sqrt{5}}$ -2+√5 |

Se pueden ahora anotar las sumas de los primeros n sumandos:

| S_1 | S_2 | S_3 | S_4 |
|-----------------|-----------------|-------|-----------------|
| $-1 + \sqrt{2}$ | $-1 + \sqrt{3}$ | 1 | $-1 + \sqrt{5}$ |

Se observa que $S_n = -1 + \sqrt{n+1}$. Para determinar el número natural para el que la suma alcanza el valor 10, hay que resolver la siguiente ecuación:

$$10 = -1 + \sqrt{n+1}$$

Esta ecuación puede resolverse con la función SOLVE, a la que se accede mediante la combinación de teclas **SHIFT** **CALC**.

(←) 1 + √ x +
1 ► ALPHA CALC 1 0 **SHIFT CALC =**

| | |
|------------------------|--|
| $-1 + \sqrt{x+1} = 10$ | $-1 + \sqrt{x+1} = 10$ $x = 120$ L-R = 0 |
|------------------------|--|

Luego, el número natural pedido es el 120.

Otra manera de resolver el problema consiste en utilizar el menú *Tabla*, para determinar la antiimagen de 10 según la función:

$$f(x) = \sum_{i=1}^x \frac{1}{\sqrt{i} + \sqrt{i+1}}$$

Para ello se procede de la siguiente manera:

SHIFT x = 1 ▼ √ x
► + √ x + 1 ►
► ► 1 ► x =

MENU 9

| | | |
|--|--|---|
| | $f(x) = \sum_{i=1}^x \left(\frac{1}{\sqrt{i} + \sqrt{i+1}} \right)$ | Rango tabla Inic.: 20 Final: 200 Paso: 10 |
|--|--|---|

Seguidamente se busca la antiimagen de 10:

| <table border="1"> <tr><th>x</th><th>f(x)</th></tr> <tr><td>1</td><td>3.5825</td></tr> <tr><td>2</td><td>4.5677</td></tr> <tr><td>3</td><td>5.4031</td></tr> <tr><td>4</td><td>6.1414</td></tr> </table> <p style="text-align: right;">20</p> | x | f(x) | 1 | 3.5825 | 2 | 4.5677 | 3 | 5.4031 | 4 | 6.1414 | <table border="1"> <tr><th>x</th><th>f(x)</th></tr> <tr><td>60</td><td>6.8102</td></tr> <tr><td>70</td><td>7.4261</td></tr> <tr><td>80</td><td>8</td></tr> <tr><td>90</td><td>8.5393</td></tr> </table> <p style="text-align: right;">90</p> | x | f(x) | 60 | 6.8102 | 70 | 7.4261 | 80 | 8 | 90 | 8.5393 | <table border="1"> <tr><th>x</th><th>f(x)</th></tr> <tr><td>9</td><td>9.0498</td></tr> <tr><td>10</td><td>9.5356</td></tr> <tr><td>11</td><td>10</td></tr> <tr><td>12</td><td>10.445</td></tr> </table> <p style="text-align: right;">120</p> | x | f(x) | 9 | 9.0498 | 10 | 9.5356 | 11 | 10 | 12 | 10.445 |
|---|--------|------|---|--------|---|--------|---|--------|---|--------|--|---|------|----|--------|----|--------|----|---|----|--------|---|---|------|---|--------|----|--------|----|----|----|--------|
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 3.5825 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 4.5677 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 5.4031 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 6.1414 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 60 | 6.8102 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 70 | 7.4261 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 80 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 90 | 8.5393 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| x | f(x) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 9 | 9.0498 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 9.5356 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 11 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | 10.445 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Como se observa, el número buscado es el 120.